$\bar{X}$ and R control charts based on marshall-olkin inverse log-logistic distribution for positive skewed data

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ABSTRACT

The fundamental assumption of variable control charts is that the data are normally distributed and spread randomly about the mean. Process data are not always normally distributed, hence there is need to set up appropriate control charts that gives accurate control limits to monitor processes that are skewed. In this study Shewhart-type control charts for monitoring positively skewed data that are assumed to be from Marshall-Olkin Inverse Log-logistic Distribution (MOILLD) was developed. Average Run Length (ARL) and Control Limits Interval (CLI) were adopted to assess the stability and performance of the MOILLD control chart. The results obtained were compared with Classical Shewhart (CS) and Skewness Correction (SC) control charts using the ARL and CLI. It was discovered that the control charts based on MOILLD performed better and are more stable compare to CS and SC control charts. It is therefore recommended that for positively skewed data, a Marshall-Olkin Inverse Log-logistic Distribution based control chart will be more appropriate.

Keywords: ARL, Control charts, CLI, Marshall-Olkin Inverse Log-logistic Distribution, Skewed distributions.

INTRODUCTION

The main purpose of Statistical Process Control (SPC) is to monitor and improve processes. It is widely applicable in both manufacturing and non-manufacturing sectors.

Shewhart control chart is an important tool employed in SPC to ensure an improved and stable process. A control chart plot is a method that assists the user in determining the best action to take on the process (Stapenhurst, 2005). The underlying assumption of control charts is that the quality characteristics are normally distributed and randomly dispersed around the mean, but there are instances where the process data are skewed and not normally distributed. This poses a major challenge using traditional method for process modeling and monitoring as the method would give inappropriate control limits when used for skewed data and thereby raise a false alarm.

Skewed process data are not normal and thereby require a predetermined model to construct a different type of Control Chart. Several heuristic methods were adopted in the literature to construct the limits in a situation where the data used in monitoring production is obtained from another distribution that are not normal (Czabak-Górska, 2017). The heuristic methods (Chang and Bai, 2001; Chan and Cui, 2003; Czabak-Górska, 2017; Atta et al., 2020) made no assumptions on the form of process distribution. However, control charting with the heuristic methods are basically adjustments of the classical Shewhart control charts.

Many authors have developed control limits for process data that are not normally distributed. Edgeman (1989) developed control charts based on Inverse Gaussian Distribution. Chakraborty and Singh (1990) developed Shewhart charts for zero-truncated Poisson distribution.

Xie and Goh (1997) studied the probability limits using geometric distribution instead of the traditional limits based on the three sigma limits. Khurshid and Chakraborty (2014) proposed Shewhart charts for zero-truncated negative binomial distribution (ZTNBD). Srinivasa et al. (2014) used percentiles of size biased Lomax distribution to construction of quality control charts. Rezac et al. (2015) developed Burr Type-XII Percentile Control Charts. Also, Srinivasa et al. (2015) developed control charts based on the evaluated percentiles of sample statistic of Weibull-Pareto distribution. Aslam et al. (2017) designed a control chart assuming that the failure time of the product follows the Weibull distribution. Wang et al. (2017) developed a monitoring scheme for the lower Weibull percentiles under complete data and Type II censoring.

Srinivasa (2018) used an attribute control chart for an exponentiated half logistic distribution. Adewara and Aako (2018) derived the control limits of variable control charts based on percentiles of exponentiated Lomax distribution. Khan et al. (2019) proposed a variable control chart under type II or failure censored reliability tests by assuming that the lifetime of a part follows the Weibull distribution with fixed and stable shape parameter. The purpose is to monitor the mean and the variance of a Weibull process. Jehan Ara et al. (2019) introduced a time-between-events chart to monitor the scheduled time assuming exponentially modified Gaussian (EMG) distribution. Burkhalter (2020) proposed Bootstrap control charts based on maximum likelihood estimator (MLE) for monitoring Pareto percentiles. Adewara et al. (2020) proposed two methods of control chart namely, Gompertz Shewhart approach and Gompertz skewness correction control charts to monitor process based on the two-parameter Gompertz distribution. Rao and Paul (2020) proposed an attribute control chart assuming that the lifetime of the manufactured goods follows a log logistic distribution. However, most of these methods are based on the percentiles distribution of the underlying distribution with predetermined inclusion probability.

This study attempts to develop control charts for variable characteristics based on Marshall-Olkin Inverse Log-logistic distributional properties for monitoring quality characteristic of a process when the normality assumption fails. Many of the data from real-life processes are always not from a normal distribution, thus, the best statistical approach is to determine the data distribution and evaluate the control limits empirically from the data (Stapenhurst, 2005).

METHODOLOGY

Marshall-Olkin Inverse Log-Logistic Distribution

Marshall-Olkin Inverse Log-logistic Distribution (MOILLD) is a two-parameter distribution derived by using Marshall-Olkin G family of distributions to generalize Inverse log-logistic distribution.

The PDF of the Inverse Log-logistic (ILLD) as defined by Para and Jan (2017) is given by

$$f(x, \gamma) = \frac{\gamma}{x^{\gamma+1}(1+x^{-\gamma})^2}; x > 0, \gamma > 0 \quad (1)$$

$$\text{while the CDF is } F(x, \gamma) = \frac{1}{1+x^{-\gamma}}; x > 0, \gamma > 0, \quad (2)$$

and its survival function is given by

$$\bar{F}(x, \gamma) = \frac{x^{-\gamma}}{1+x^{-\gamma}}; x > 0, \gamma > 0 \quad (3)$$

where γ is the shape parameter.

The survival function of Marshall-Olkin G family of distribution is given by

$$\bar{G}(x) = \frac{\alpha \bar{F}(x)}{1 - \bar{\alpha} \bar{F}(x)}; \text{ where } \alpha > 0, \bar{\alpha} = 1 - \alpha \text{ and } -\infty < x < \infty. \quad (4)$$

and the PDF is

$$g(x) = \frac{\alpha f(x)}{(1 - \bar{\alpha} \bar{F}(x))^2} \quad (5)$$

where $\alpha > 0, \bar{\alpha} = 1 - \alpha$ and $-\infty < x < \infty$.

Marshall-Olkin Inverse Log-logistic Distribution (MOILLD) is derived by inserting Equation (3) in Equation (4) to get the survival function and Equations (1) and (3) in Equation (5) to get the corresponding density function.

The survival function of MOILLD is given by

$$\bar{G}(x) = \frac{\alpha x^{-\gamma}}{(1+x^{-\gamma}) - (1-\alpha)x^{-\gamma}}.$$

Therefore,

$$\bar{G}(x) = \frac{\alpha x^{-\gamma}}{1 + \alpha x^{-\gamma}}; x > 0, \gamma > 0, \alpha > 0. \quad (6)$$

The CDF is given by

$$G(x) = 1 - \bar{G}(x),$$

substituting $\bar{G}(x)$ from equation (6),

$$G(x) = 1 - \frac{\alpha x^{-\gamma}}{1 + \alpha x^{-\gamma}}$$

$$G(x) = \frac{1}{1 + \alpha x^{-\gamma}}; \quad x > 0, \gamma > 0, \alpha > 0. \quad (7)$$

While the PDF of MOILLD is given by

$$g(x) = \frac{\alpha \gamma}{x^{\gamma+1} (1 + \alpha x^{-\gamma})^2}; \quad x > 0, \gamma > 0, \alpha > 0. \quad (8)$$

Below are the shapes of the PDF of MOILLD for selected values of the parameters α and γ in Figure 1. In the first graph of Figure 1, the two parameters of graph were varied to determine the shape of the distribution. While, in the second graph, only the shape parameter (γ) was varied.

Figure 1. Probability Density Plots of MOILLD

The result of Figure 1 above revealed that MOILLD is unimodal and positively skewed. The skewness tends to zero as the parameters increase simultaneously.

Mean and Variance of MOILLD

Let X be a random variable that has the MOILLD, then, the r th non-central moments is given by

$$E(X^r) = \int_0^{\infty} x^r g(x) dx$$

where $g(x)$ is as given in equation (8).

Therefore,

$$E(X^r) = \alpha^{\frac{r}{\gamma}} \frac{\pi r}{\gamma} \csc\left(\frac{\pi(\gamma-r)}{\gamma}\right); \quad r > 0, \gamma > 0, \alpha > 0. \quad (9)$$

where $\csc(\cdot)$ is the cosecant function.

The first two moments ($r=1$ and $r=2$) about the origin for MOILLD are given by

$$E(X) = \alpha^{\frac{1}{\gamma}} \frac{\pi}{\gamma} \csc\left(\frac{\pi(\gamma-1)}{\gamma}\right) \quad (10)$$

and

$$E(X^2) = 2\alpha^{\frac{2}{\gamma}} \frac{\pi}{\gamma} \csc\left(\frac{\pi(\gamma-2)}{\gamma}\right). \quad (11)$$

Thus, the mean is given by equation (10). Similarly, the variance of MOILLD using Equations (10) and (11) is given by

$$\begin{aligned} \text{Var}(X) &= E(X^2) - [E(X)]^2 \\ &= 2\alpha^{\frac{2}{\gamma}} \frac{\pi}{\gamma} \csc\left(\frac{\pi(\gamma-2)}{\gamma}\right) - \left(\alpha^{\frac{1}{\gamma}} \frac{\pi}{\gamma} \csc\left(\frac{\pi(\gamma-1)}{\gamma}\right)\right)^2 \\ &= \alpha^{\frac{2}{\gamma}} \frac{\pi}{\gamma} \left(2 \csc\left(\frac{\pi(\gamma-2)}{\gamma}\right) - \frac{\pi}{\gamma} \csc^2\left(\frac{\pi(\gamma-1)}{\gamma}\right)\right). \end{aligned} \quad (12)$$

Derivation of Range

The distribution of range is computed using the difference between the pdf of maximum and minimum order statistics of MOILLD in order to compute the range statistic.

Let $X_{(1)}, X_{(2)}, X_{(3)}, \dots, X_{(n)}$ be the order statistics from the sample $X_1, X_2, X_3, \dots, X_n$ of size n from a continuous population with CDF $G(X)$ as given in equation(7) and PDF $g(X)$ as given in equation (8). then the pdf of r th order statistics $X_{(r)}$ is given by

$$g_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} g(x) [G(x)]^{r-1} [1 - G(x)]^{n-r}; \quad r = 1, 2, \dots, n.$$

Therefore, pdf of the maximum order statistic $X_{(n)}$ is

$$g_{x(n)}(x) = \frac{n\alpha\gamma}{x^{\gamma+1} (1 + \alpha x^{-\gamma})^2} \left[\frac{1}{1 + \alpha x^{-\gamma}} \right]^{n-1}. \quad (13)$$

and the pdf of the least order statistic $X_{(1)}$ is

$$g_{x(1)}(x) = \frac{n\alpha\gamma}{x^{\gamma+1} (1 + \alpha x^{-\gamma})^2} \left[\frac{\alpha x^{-\gamma}}{1 + \alpha x^{-\gamma}} \right]^{n-1}. \quad (14)$$

Hence, the range of the distribution is

$$R(x) = g_{x(n)}(x) - g_{x(1)}(x)$$

After simplification,

$$R(x) = \left(\frac{n\alpha\gamma}{x^{\gamma+1}(1+\alpha x^{-\gamma})^{n+1}} \right) (1 - [\alpha x^{-\gamma}]^{n-1});$$

$$x > 0, \gamma > 0, \alpha > 0$$

$$\therefore R(x) = \frac{n\alpha\gamma(1-[\alpha x^{-\gamma}]^{n-1})}{x^{\gamma+1}(1+\alpha x^{-\gamma})^{n+1}}; x > 0, \gamma > 0, \alpha > 0.$$

(15)

Moments of R(x)

Let X_j ($j = 1, \dots, n$) be range of sample size n from MOILLD, then the non-central moments of the distribution of range is given by

$$E(x_j^r) = \int_0^\infty x_j^r R(x) dx$$

$$= \int_0^\infty x_j^r \left(\frac{n\alpha\gamma(1-[\alpha x^{-\gamma}]^{n-1})}{x^{\gamma+1}(1+\alpha x^{-\gamma})^{n+1}} \right) dx.$$

After simplification, we have

$$E(x_j^r) = - \frac{\sin\left(\frac{\pi(r+\gamma)}{\gamma}\right) \Gamma\left(\frac{r+\gamma}{\gamma}\right)^2 \Gamma\left(\frac{\gamma(n-r)}{\gamma}\right) \gamma + \Gamma\left(\frac{\gamma(n-r)}{\gamma}\right) \pi r \alpha^{r/\gamma}}{\Gamma(n) \gamma \sin\left(\pi \frac{r+\gamma}{\gamma}\right) \Gamma\left(\frac{r+\gamma}{\gamma}\right)}. \quad (16)$$

where $\sin(\cdot)$ is the sine function. The first two moments about the origin for the range of MOILLD are respectively given by

$$E(R) = - \frac{\sin\left(\frac{\pi(1+\gamma)}{\gamma}\right) \Gamma\left(\frac{1+\gamma}{\gamma}\right)^2 \Gamma\left(\frac{\gamma(n-1)}{\gamma}\right) \gamma + \Gamma\left(\frac{\gamma(n-1)}{\gamma}\right) \pi \alpha^{1/\gamma}}{\Gamma(n) \gamma \sin\left(\pi \frac{1+\gamma}{\gamma}\right) \Gamma\left(\frac{1+\gamma}{\gamma}\right)}.$$

(17)

$$E(R^2) = - \frac{\sin\left(\frac{\pi(2+\gamma)}{\gamma}\right) \Gamma\left(\frac{2+\gamma}{\gamma}\right)^2 \Gamma\left(\frac{\gamma(n-2)}{\gamma}\right) \gamma + 2\Gamma\left(\frac{\gamma(n-2)}{\gamma}\right) \pi \alpha^{2/\gamma}}{\Gamma(n) \gamma \sin\left(\pi \frac{2+\gamma}{\gamma}\right) \Gamma\left(\frac{2+\gamma}{\gamma}\right)}.$$

(18)

Thus, the variance of the range of MOILLD is given by

$$Var(R) = E(R^2) - [E(R)]^2. \quad (19)$$

Maximum Likelihood Estimation of MOILLD

Parameters

The two parameters to be estimated are α and γ . The maximum likelihood estimation (MLE) method is used to obtain estimates of the parameters. Thus, if $X_1, X_2, X_3, \dots, X_n$ is a variable of size n from MOILLD, then the

likelihood function of the distribution is

$$L(x|\alpha, \gamma) = \prod_{i=1}^n \frac{\alpha\gamma}{x_i^{\gamma+1}(1+\alpha x_i^{-\gamma})^2};$$

$$x > 0, \gamma > 0, \alpha > 0. \quad (20)$$

By taking logarithm of Equation (20), we find the log likelihood function

$$l = \log L(x|\alpha, \gamma) = n \log(\alpha\gamma) - (\gamma+1) \sum_{i=1}^n \log(x_i) - 2 \sum_{i=1}^n \log(1 + \alpha x_i^{-\gamma}). \quad (21)$$

The MLE's of α and γ is obtained by differentiating the log likelihood with respect to α and γ

Thus, we have

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} - \frac{2n x_i^{-\gamma}}{1 + \alpha x_i^{-\gamma}}. \quad (22)$$

$$\frac{\partial l}{\partial \gamma} = \frac{n}{\gamma} - n \ln(x_i) + \frac{2n \alpha x_i^{-\gamma} \ln(x_i)}{1 + \alpha x_i^{-\gamma}}. \quad (23)$$

setting Equations (22) and (23) to zero, we have equations (24) and (25)

$$\frac{n}{\alpha} - \frac{2n x_i^{-\gamma}}{1 + \alpha x_i^{-\gamma}} = 0 \quad (24)$$

and

$$\frac{n}{\gamma} - n \ln(x_i) + \frac{2n \alpha x_i^{-\gamma} \ln(x_i)}{1 + \alpha x_i^{-\gamma}} = 0. \quad (25)$$

The two derivative equations in (24) and (25) cannot be solved analytically; hence, we resorted to numerical solution using Newton-Raphson method.

Variable Control Charts based on MOILLD

Let X_{ij} be a random variable of size n obtained from m subgroup, $i = 1, 2, \dots, n$ while $j = 1, 2, \dots, m$. If the underlying distribution is normal, the 3-sigma limits for R-chart are used and these are available in any good textbook (Montgomery, 2012).

When the sample data is not normal and is assumed to be independent from MOILLD, the control limits for R-chart for 3-sigma is of the form

$$\begin{cases} UCL = E(R) + 3 \frac{\sqrt{Var(R)}}{\sqrt{n}} \\ CL = E(R) \\ LCL = E(R) - 3 \frac{\sqrt{Var(R)}}{\sqrt{n}} \end{cases}. \quad (26)$$

where $E(R)$, and $\sqrt{Var(R)}$, are as derived in Equations (17) and (19).

Also, the 3-sigma control limits for the $\bar{X}$ chart are;

$$\begin{cases} UCL = E(X) + \frac{3\sqrt{Var(\bar{X})}}{c_4\sqrt{n}} \\ CL = E(X) \\ LCL = E(X) - \frac{3\sqrt{Var(\bar{X})}}{c_4\sqrt{n}} \end{cases} \quad (27)$$

where $\bar{X}$ is estimated by $E(X)$ as in Equation (10) and σ is estimated by $\frac{\sqrt{Var(\bar{X})}}{c_4}$, $Var(X)$ as in Equation (12).

Let $A_3 = \frac{3}{c_4\sqrt{n}}$, then Equation (27) reduces to:

$$\begin{cases} UCL = E(X) + A_3\sqrt{Var(X)} \\ CL = E(X) \\ LCL = E(X) - A_3\sqrt{Var(X)} \end{cases} \quad (28)$$

The value of the constants A_3 can obtain from a Quality control table.

Performance Evaluation

The control charts performance is evaluated using Control Limit Interval (CLI) and Average Run Length (ARL). The CLI is the difference between the control limits values. Therefore, for the R-control charts, the control limit interval is given by the expression:

$$CLI_s = 6 \frac{\sqrt{Var(R)}}{\sqrt{n}} \quad (29)$$

Similarly, the control limit interval for the $\bar{X}$ control chart is given by the expression:

$$CLI_x = 2A_3\sqrt{Var(X)} \quad (30)$$

A control chart will be declared superior if its CLI is the smallest out of all the control charts under consideration.

The Average Run Length (ARL) is the expected number of subgroups until an out of control signal. It is the reciprocal of the type I error rate (α). A good control chart is expected to have minimal type I error or false alarm rate. Given the

false alarm rate α , where α is defined as,

$$\alpha = P(x < LCL) + P(x > UCL)$$

Then the average run length is given by $ARL = \frac{1}{\alpha}$. It is axiomatic that for any Shewhart chart having the 3σ limit, 99.73% of the quality characteristic falls within the control limits. The smaller the ARL, the better the control charting method in detecting out-of-control.

RESULTS AND DISCUSSION

Generated Data from MOILLD ($\alpha = 4.5, \gamma = 5.8$)

The data for this study is obtain from a MOILLD with $\alpha = 4.5$ and $\gamma = 5.8$. Let $X_n, n=1,2, \dots$ be a random variable of size n . Using the control limits obtained in Equations (26) and (28) for the MOILLD charts and the limits of the CS and SC method in the literature. $\bar{X}$ and R charts for MOILLD were evaluated using a data set generated and arranged in 3, 4, 5, 7 and 10 sample sizes with 30 subgroups. R programming language was used to obtain the limits, the CLI and ARL for MOILLD charts. The CS and SC were used to monitor the data generated for comparison. The summary for the three approaches is presented in Table 1 below.

Table 1. ARL and CLI for CS, SK and MOILLD Control Charts (MC)

Sample Size	Method	$\bar{X}$ Charts		R Charts	
		CLI	ARL	CLI	ARL
3	CS	2.0472	30	2.5755	∞
	SC	2.0880	∞	3.2819	∞
	MC	1.4133	10	2.9731	∞
4	CS	1.9950	∞	3.1224	30
	SC	2.0024	∞	3.7354	30
	MC	1.2716	7.5	3.1131	30
5	CS	1.7383	∞	3.1844	30
	SC	1.7728	∞	3.6755	30
	MC	0.9288	6	2.7405	30
7	CS	1.5272	30	3.3679	15
	SC	1.8127	∞	4.4468	∞
	MC	0.7719	3.3	2.6562	10
10	CS	1.3322	∞	3.3607	10
	SC	1.3453	∞	3.4602	7.5
	MC	1.5246	2.7	2.2835	2.7

The results of the generated data presented in Table 1 showed that MOILLD $\bar{X}$ control charts have lowest CLI and ARL compared to CS and SC limits irrespective of the sample size. R charts based on MOILLD have the lowest CLI for all sample sizes except when $n=3$, lowest ARL when $n = 7$ and 10, and equal ARL when $n = 3, 4$ and 5 compared to CS and SC limits irrespective of the sample size. R charts based on MOILLD have the lowest CLI for

all sample sizes except when $n=3$, lowest ARL when $n = 7$ and 10, and equal ARL when $n = 3, 4$ and 5.

Data on Asthma Patients' Admission

The data on asthma patients was collected from Asthmatic in-patients in a Hospital in Nigeria. 185 patients' data were retrieved and grouped into 37 subgroups of 5 units as presented in Table 2. The control charts were developed to monitor the patience and to assess the performance.

To explore the data, the Jarque Bera Test for normality was calculated to be 39.908 with a p-value of $2.2e-16$. The skewness was 1.1257; this showed that the data was positively skewed, and thus, not normally distributed. The Q-Q plots of the data were presented in Figure 2 below to further strengthen the earlier submission that the data is highly positively skewed.

Table 2. Asthma Patients' Length of Stay on Admission

M	X_1	X_2	X_3	X_4	X_5
1	2	66	23	7	27
2	1	30	42	7	55
3	12	23	24	7	66
4	10	22	52	2	44
5	32	2	28	22	12
6	3	17	37	7	2
7	4	2	49	6	1
8	56	3	3	33	32
9	63	13	4	7	43
10	51	23	3	7	2
11	3	22	6	7	65
12	34	56	4	21	1
13	42	4	45	2	21
14	65	7	2	2	3
15	55	4	6	3	4
16	66	8	6	1	5
17	6	12	7	45	42
18	1	4	9	33	61
19	3	8	29	38	6
20	1	9	22	33	21
21	2	2	2	42	41
22	5	48	2	45	23
23	7	7	3	3	32
24	72	7	7	45	71
25	23	55	3	65	22
26	21	3	4	5	33
27	45	67	5	6	52
28	44	16	15	12	3
29	42	6	7	8	4
30	42	3	7	21	5
31	10	12	7	42	11
32	12	8	7	33	65
33	2	5	6	5	9
34	1	4	6	4	1
35	1	8	87	4	10
36	4	43	7	6	13
37	4	1	7	7	10

The empirical cumulative density function (ecdf) plot of the data with hypothetical MOILLD presented in Figure 3 confirmed that the data can be model with the MOILLD. Hence, the developed MOILLD control chart is appropriate for monitoring the mean and spread of the data on asthma patients' length of stay.

Computation of Control Limits

From the data presented in Table 2, the mean and range of the asthma patients' length of stay on admission to hospital were computed together with the control limits for the CS and SC methods. Also, MOILLD was fitted to the data and its parameters estimated using maximum likelihood estimation method. The distribution's parameters were used to calculate the statistics needed for the construction of control limits for MOILLD charts. Results of CLI and ARL for the three methods were presented in Table 3.

The results of real-life data presented in Table 3 revealed that MOILLD charts have the lowest CLI for the mean and range control charts. However, the ARL for the mean charts are infinitely large while the ARL for the range under Skewness Correction method is less than the ARL of the MOILLD.

Figure 2. Histogram and Q-Q plots of the Asthma Patients' length of stay

Figure 3. CDF of the Data and MOILLD

Table 3. Control Limits Interval and Average Run Length of Asthma Patients' Length of Stay

Method	$\bar{X}$ Charts		R Charts	
	CLI	ARL	CLI	ARL
CS	52.9281	∞	96.9583	∞
SC	53.9774	∞	111.9103	18.5
MC	46.6354	∞	80.5544	37

DISCUSSION

The performance indices result for the generated MOILLD data presented in Table 1 for MOILLD mean charts showed that the CLI of the charts perform better with lowest values compared to CS chart and SC control chart proposed by Chan and Cui (2003) irrespective of the sample size. Also, the ARL of the $\bar{X}$ chart revealed that the proposed model performed better than the existing models.

The R charts based on MOILLD for the generated data was also considered. The results revealed that at the CLI of sample size 3, the existing charts were better but as the sample size increases, the proposed chart CLI performed better for sample sizes 4, 5, 7 and 10.

The ARL for R chart results showed that proposed model was not fit for sample size 3. As the sample size increases to 4 and 5, the ARL was the same. Finally, as the sample size increases to 7 and 10, the proposed chart performs better than the existing control charts used in this study. Similarly, the real-life data results presented in Table 2 revealed that MOILLD charts have the lowest CLI for the mean and range control charts. However, the ARL for the range under SC method is less than the ARL of the MOILLD. In order to investigate further, we compared the results from the generated data in Table 1 for $n = 5$, with the results in Table 3 for the range. It was discovered that the ARL for the three methods are equal. This is an indication that SC method can raise false alarm. MOILLD charts performed well for small and large sample sizes.

The results of MOILLD charts are in line with the results of Adewara et al. (2020) where Gompertz based $\bar{X}$ charts performed better than charts based on Skewness Correction (SC) method. Hence, the control charts based on the distributional properties of underlying distribution perform better than control charts based on adjustments of the classical Shewhart control charts.

CONCLUSION

This study derived the pdf of MOILLD and uses its distributional properties to develop Shewhart-type Control Charts for skewed and heavily tailed distributions. A simulation study was carried out to compare the performance and stability of the proposed charts with CS and SC control charts using CLI and ARL as performance indices. The results showed that control charts based on MOILLD performed better and are stable compare to CS control chart and SC control chart. The real-life data result revealed that the MOILLD approach is able to detect out-of-control faster and can easily raise false alarm if there exist any out of control otherwise it remains in control. Hence, it can be concluded that when data are positively skewed and follows a MOILLD, the developed charts will be more appropriate for monitoring the mean and spread of the data.

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