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A Standard Deviation Control Chart for the Marshall Olkin Inverse Log-logistic Distribution

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Abstract

Most process data in real life applications are positively skewed distribution, thereby the traditional Shewhart-*S* control chart with the assumption of normality may not be appropriate. A control chart for tracking the dispersion of data from Marshall Olkin Inverse Log-logistic distribution (MOILLD) is presented in this study. The performance of the proposed control chart is compared with control charts based on the Shewhart, Skewness Correction (SC), Weighted Standard Deviation (WSD), and Median Absolute Deviation (MAD) using simulation study. The simulation results demonstrate that when skewed data are from MOILLD, the proposed control chart detects out-of-control points faster than existing methods and therefore, outperforms the existing control standard deviation control charts. The proposed control chart was further applied to a real-life skewed data and the obtained results aligned with the simulated results. Therefore, it is recommended that users in the manufacturing and industry sectors implement the proposed approach, particularly in cases where the process data exhibit positive skewness.

Keywords: Control charts, Dispersion, MOILLD, Monitoring, Skew data.

Introduction

The study of important process variables and the detection of quality enhancements or degradations can both be accomplished with the help of statistical process control (SPC). Control charts are furthermost and effective SPC tools for monitoring and learning about a process to gradually improve it. The most popular technique for keeping track of process variability is the Shewhart-*S* control chart. The primary goal of control charting is to identify assignable causes when they occur so that the appropriate corrective action(s) can be taken before a significant amount of non-conforming product is produced (Chou et al., 2001). The quality parameter of interest must follow a normal distribution in order to use Shewhart-S charts, which are employed in industry and by professionals in the field of quality. However, in actual corporate settings and hospitals, process distributions deviate from normalcy, making it risky to infer from typical control charts that a process is stable or capable of meeting consumer demands. The majority of process data are big and highly skewed and are therefore abnormal. The population of the process does not conform to the normalcy assumption, and it may be deceptive to monitor the process dispersion using the Shewhart standard deviation chart. Hence, a non-normal control chart based on the underlying distribution must be developed.

Several authors have developed control chart limits methods for monitoring skewed data. Control charts were developed by Chang and Bai (2001) for populations with positively skewed standard deviations. Chan and Cui (2003) developed SC $\overline{X}$ and R charts to monitor skewed distribution. The MAD was used by Adekeye and Azubuike (2012) to determine the limits for control charts used to monitor non-normal processes. Shibo et al. (2014) developed a control chart with a WSD. Atta et al. (2020) presented a SC-S chart that was specifically designed for monitoring process dispersion in skewed distributions. Using the scaled WSD method, Atta et al. (2020) developed an improved R control chart for monitoring process dispersion in skewed populations. Aako et al. (2020a) created $\overline{X}$ and R Control Charts to monitor positively skewed MOILLD data. Aako et al. (2020_b) proposed robust scale estimator-based control charts for the MOILLD. The Gompertz Shewhart technique and the Gompertz SC control charts are two control chart methods that Adewara et al. (2020) suggested for monitoring the process based on Gompertz distribution. The efficiency of dispersion control charts with skewed distributions was examined by Ying-Chin et al. in 2021. Wan and Zhu (2022) suggested an enhanced joint $\overline{X}$ and R chart to monitor the process location and dispersion.
In this study, a control chart for tracking the dispersion of a process presumed to be from the MOILLD's is proposed. The data will be modeled using MOILLD, and the distribution parameters would be estimated, which would then be used to evaluate the statistic needed for the proposed S control chart.

2. MOILLD
The probability density function (pdf) and cumulative distribution function (cdf) of the MOILLD as defined by Aako et al. (2022) are respectively

$$g(x) = \frac{\alpha\gamma}{x^{\gamma+1}(1+\alpha x^{-\gamma})^2}; \quad ¿0, \ \gamma > 0, \ \alpha > 0, \qquad (1)$$

and

$$G(x) = \frac{1}{1+\alpha x^{-\gamma}}; \quad ¿0, \ \gamma > 0, \ \alpha > 0, \qquad (2)$$

and the survival function is

$$\bar{G}(x) = \frac{\alpha x^{-\gamma}}{1+\alpha x^{-\gamma}}; \ x > 0, \gamma > 0, \alpha > 0 . \quad (3)$$

where γ is the shape parameter.

Thus, the mean is defined as

$$E(X) = \alpha^{\frac{1}{\gamma}} \frac{\pi}{\gamma} \csc\left(\frac{\pi(\gamma-1)}{\gamma}\right) \quad (4)$$

and the variance is defined as

$$Var(X) = \alpha^{\frac{2}{\gamma}} \frac{\pi}{\gamma}\left(2 \csc\left(\frac{\pi(\gamma-2)}{\gamma}\right) - \frac{\pi}{\gamma}\csc^2\left(\frac{\pi(\gamma-1)}{\gamma}\right)\right). \quad (5)$$

2.1 Proposed Standard Deviation Control Chart for the MOILLD

Let X_{ij} be a random sample of size n selected from a subgroup of size m, with $i=1,2,\ldots,n$ and $j=1,2,\ldots,m$. If the underlining distribution is normal, then $C_4\sigma$ is an unbiased estimator of *S*.

$C_4 = \left(\frac{2}{n} - 1\right)^{\frac{1}{2}} \left[\frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}\right]$ and the standard deviation of S is $\sigma\sqrt{1 - C_4^2}$.

Therefore, the 3σ control limits for *S*- chart is

$$\begin{cases} UCL = C_4\sigma + 3\sigma\sqrt{1 - C_4^2} \\ CL = C_4\sigma \\ LCL = C_4\sigma - 3\sigma\sqrt{1 - C_4^2} \end{cases} \quad (6)$$

Standard deviation can be calculated using MOILLD parameters when the distribution is skewed by using (5).

As a result, when σ in (6) is substituted by $\sqrt{Var(X)}$, we have

$$\begin{cases} UCL = C_4\sqrt{Var(X)} + 3\sqrt{Var(X)}\sqrt{1 - C_4^2} \\ CL = C_4\sqrt{Var(X)} \\ LCL = C_4\sqrt{Var(X)} - 3\sqrt{Var(X)}\sqrt{1 - C_4^2} \end{cases} \quad (7)$$

Substitute (5) into (7), then the Upper Control Limit (UCL) for the proposed control chart becomes

$$UCL = C_4 \sqrt{\alpha^{\frac{2}{\gamma}} \frac{\pi}{\gamma}\left(2\csc\left(\frac{\pi(\gamma-2)}{\gamma}\right) - \frac{\pi}{\gamma}\csc^2\left(\frac{\pi(\gamma-1)}{\gamma}\right)\right)}$$

$$+3\sqrt{= \alpha^{\frac{2}{\gamma}} \frac{\pi}{\gamma}\left(2\csc\left(\frac{\pi(\gamma-2)}{\gamma}\right) - \frac{\pi}{\gamma}\csc^2\left(\frac{\pi(\gamma-1)}{\gamma}\right)\right)}\sqrt{1-C_4^2}$$

$$= \sqrt{\alpha^{\frac{2}{\gamma}} \frac{\pi}{\gamma}\left(2\csc\left(\frac{\pi(\gamma-2)}{\gamma}\right) - \frac{\pi}{\gamma}\csc^2\left(\frac{\pi(\gamma-1)}{\gamma}\right)\right)} \left(C_4 + 3\sqrt{1-C_4^2}\right) \quad (8)$$

and the Lower Control Limit (LCL) becomes

$$LCL = \sqrt{\alpha^{\frac{2}{\gamma}} \frac{\pi}{\gamma}\left(2\csc\left(\frac{\pi(\gamma-2)}{\gamma}\right) - \frac{\pi}{\gamma}\csc^2\left(\frac{\pi(\gamma-1)}{\gamma}\right)\right)} \left(C_4 - 3\sqrt{1-C_4^2}\right) \quad (9)$$

where csc(.) is the Cosecant function.

2.2 Performance Evaluation

To assess the effectiveness of the suggested control charts, the Control Limit Interval (CLI), Average Run Length (ARL), and Standard Deviation Run Length (SDRL) are employed. The indices' computation formulas are as described by Abu-Shawiesh et al (2020).

3.0 Results and Discussion

3.1 Simulation Study

The effectiveness of the suggested S control chart was evaluated using data sets generated for $n=3,4,5,7,$and 10 with 30 subgroups using the MOILLD with parameters ($\alpha = 2.5, \gamma = 1.5$), ($\alpha = 3.5, \gamma = 2.8$), and ($\alpha = 4.5$, $\gamma = 5.8$). The Standard deviation control limits were calculated using Eq (8) for the UCL, and Eq (9) for the LCL for the generated data in different sample sizes and compared to existing techniques such as Shewhart (SH), Skewness Correction (SC), Weighted Standard Deviation (WSD), and Median Absolute Deviation (MAD). The R programming language was used to calculate the CLI, ARL and SDRL for the five techniques. The obtained CLI, ARL and SDRL for the methods are presented in Table 1.

Table 1: CLI, ARL and SDRL of SH, SC, MC, WSD and MAD Control Charts.

n	Method	$\alpha = 2.5, \gamma = 1.5$			$\alpha = 3.5, \gamma = 2.8$			$\alpha = 4.5, \gamma = 5.8$		
		CLI	*ARL*	*SDRL*	*CLI*	*ARL*	*SDRL*	*CLI*	*ARL*	*SDRL*
n=3	SH	2.56	1.80	0.89	1.80	1.40	0.86	1.45	1.10	0.83
	SC	6.71	25.43	5.00	3.58	23.24	7.30	2.51	21.17	8.67
	MC	6.09	0.56	0.80	3.42	0.71	0.96	2.41	0.79	1.09
	WSD	NaN	*NA*	*NA*	1.57	2.04	0.94	1.19	2.08	0.97
	MAD	2.18	0.61	0.65	1.74	0.43	0.58	1.49	0.37	0.54
n=4	SH	2.40	2.12	0.95	1.67	1.61	0.90	1.34	1.27	0.84
	SC	6.52	25.43	5.12	3.30	23.09	7.55	2.29	20.78	9.15
	MC	3.94	1.50	1.38	1.75	2.48	2.06	1.19	3.17	2.54
	WSD	NaN	*NA*	*NA*	1.47	2.32	0.94	1.12	2.32	1.00
	MAD	1.88	0.61	0.66	1.51	0.36	0.54	1.29	0.28	0.49
n=5	SH	2.31	2.35	1.00	1.60	1.77	0.95	1.27	1.41	0.89
	SC	6.45	25.29	5.42	3.17	23.13	7.41	2.17	20.88	9.13
	MC	2.89	2.65	1.98	1.28	4.81	3.35	0.87	6.55	4.15
	WSD	NaN	*NA*	*NA*	1.41	2.54	1.02	1.08	2.52	1.04
	MAD	1.65	1.07	0.83	1.34	0.86	0.76	1.16	0.75	0.74
n=7	SH	2.07	2.75	1.10	1.40	2.06	1.00	1.11	1.61	0.94
	SC	6.32	25.38	5.31	2.97	22.84	7.72	1.95	20.60	9.28
	MC	1.80	5.36	3.21	0.87	9.85	5.14	0.59	13.56	5.91
	WSD	NaN	*NA*	*NA*	1.34	2.86	1.11	1.02	2.80	1.11
	MAD	1.37	1.39	1.00	1.13	1.15	0.94	0.97	1.03	0.93
n=10	SH	1.76	3.61	1.64	1.18	2.45	1.14	0.93	1.87	1.04
	SC	5.95	25.03	5.63	2.74	22.62	7.88	1.77	20.37	9.36
	MC	1.29	9.10	4.58	0.60	16.20	6.36	0.41	21.09	5.86
	WSD	NaN	29	0	1.13	28	0	0.88	0	0
	MAD	1.08	1.73	1.21	0.90	1.38	1.09	0.77	1.26	1.06

Table 1 presents the CLI, ARL and SDRL of SH, SC, MC, WSD and MAD Control Charts for different sample sizes. The CLI of the five methods decreases and the ARL of SH and MAD decrease while that of MC and WSD increase as the parameters increase simultaneously. Also, CLI decreases as sample size increases. ARL and SDRL of SC method remain almost the same as n increases. The ARL and SDRL of SH and MC methods increase as n increase while that of MAD decreases as n increases. WSD and MAD become inappropriate as n increases. MC method has the smallest CLI especially when $n \geq 5$ for different parameters. It can be concluded that SC, WSD and MAD methods could raise a false alarm, hence, MC method would be more appropriate and detect out-of-control faster than the competing methods.

3.2 Real Data Application

The performance of the proposed control chart was evaluated using the data on the thickness of an oil seal. The data consist of thirty subgroups of five data sets each were created from 150 data sets (see Table 2). The proposed control chart was used to determine whether an oil seal's thickness is outside the production process's control parameters.

Table 2: The thickness of oil seals data.

X_1	X_2	X_3	X_4	X_5
1.9	1.9	1 .8	1.9	1.9
2 .0	1 .8	1.9	1.9	2 .0
2 .2	2 .1	2 .0	2.3	1 .8
2 .0	2.4	2 .1	2.5	2 .0
2 .0	2 .0	2.4	2 .0	2 .1
1.9	2.3	1 .6	1.7	1.9
1.7	2.3	1 .6	1.7	1 .8
2 .0	2.3	1 .8	1 .8	1 .6
2 .0	2.3	1.9	2 .2	1 .6
2 .0	2 .2	2 .0	2 .2	2 .2
2 .1	1 .8	2 .1	2.3	2 .1
2 .1	1 .6	1 .8	2 .0	2 .1
2.4	1 .8	1 .8	2 .0	2 .0
2 .2	1 .8	1.9	1.9	2 .0
2 .2	2 .1	2 .2	1 .8	1.9
2.4	2 .2	2 .0	2.3	2 .0
2 .2	2 .2	2	2 .1	1.9
1 .8	2.4	1 .8	2 .2	2 .0
2 .0	1 .8	2 .0	1.9	2 .0
2 .1	1 .6	1 .6	1 .8	2.4
1.9	1 .8	2 .1	1.8	2.4
2 .1	2 .0	1.7	2 .1	1.9

1.9	2 .2	2.3	2 .1	1.9
2.4	2.3	1.7	2 .0	2.4
1 .8	2.3	2.4	2.4	1.9
2.4	2 .2	1.9	1 .8	1 .8
1.9	2 .0	1.9	1 .6	1 .8
1 .8	1 .8	2	2 .2	2 .2
2 .2	1 .8	2 .1	2.3	1 .8
1.7	2 .0	2 .0	2 .0	1 .8

Figure 1 shows the density and $Q-Q$plots of the data to help determine the shape of the data distribution. Also, the Jarque Bera Test for normality was 1.1242, with a p-value of 0.41. Furthermore, the skewness coefficient for the data was 0.0877. The results show that the data is not dispersed regularly and there is evidence that the data is from a positively skewed distribution.

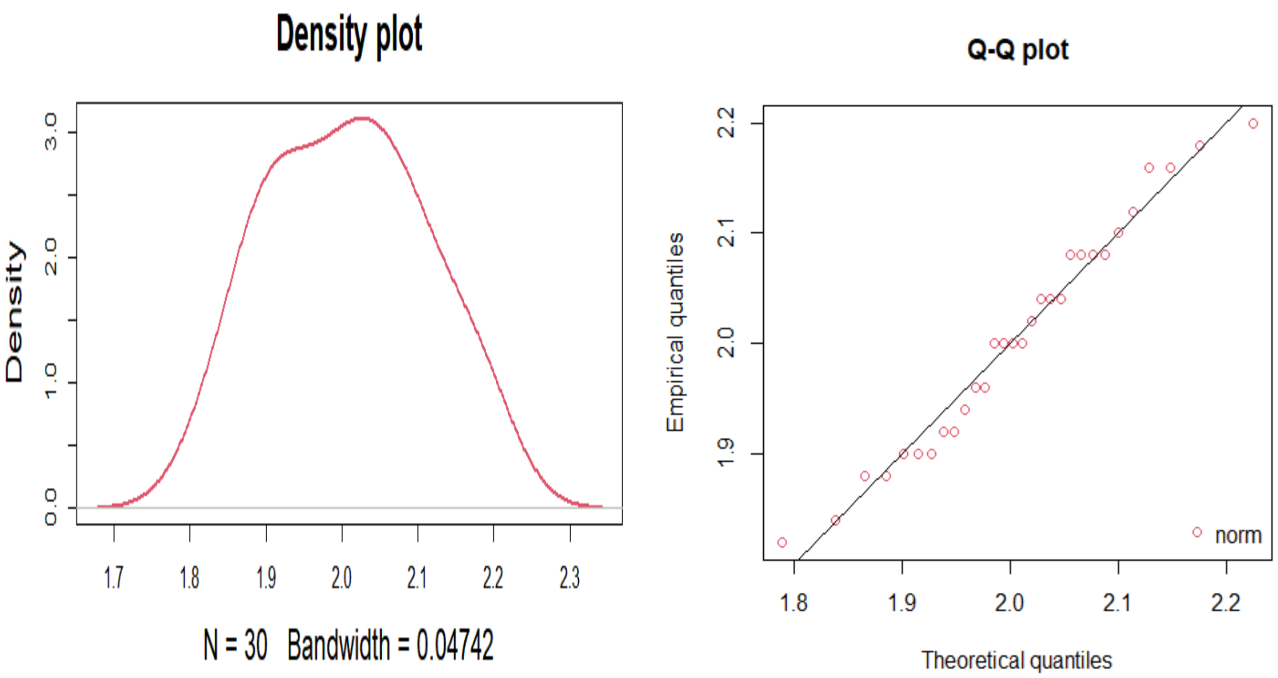

Figure 1: *Density* $\wedge$ *Q* − *Q plots of Thickness of Oil seal Data*

Based on the fact that the data is a skewed data, there is need to use appropriate non-normal approach to construct the control limits that will not raise false alarm. MOILLD was used to model the data and the graphical representation is presented in Figure 2. Therefore, it is clear that the data is approximately a Marshall-Olkin Inverse Log-logistic random variable.

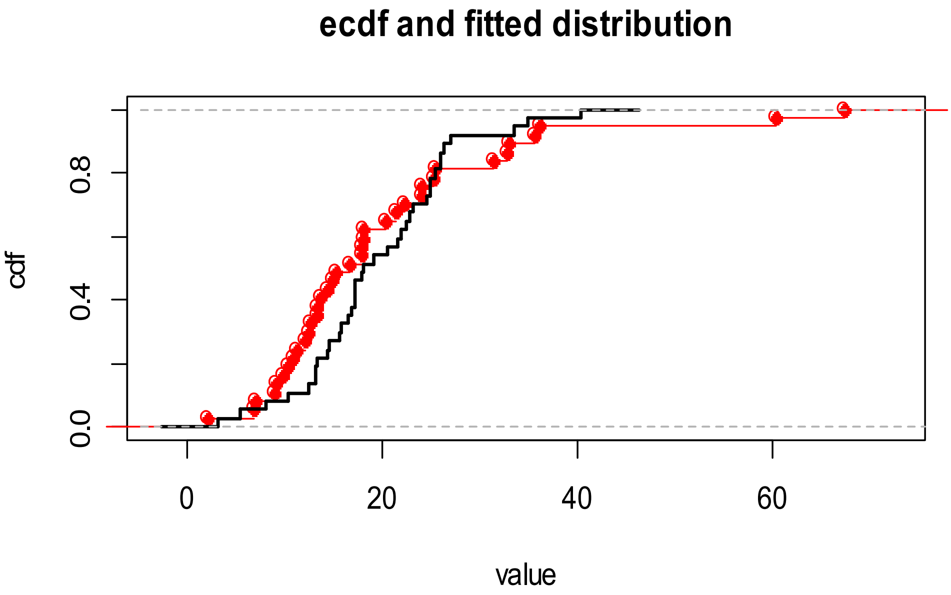

Figure 2: *CDF of Thickness of Oil seal Data* ∧ *Hypothetical MOILLD* .

The $\overline{X}$ and S of the oil seal thickness data were calculated and utilized to develop control limits for the proposed charts. Table 3 shows the CLI and ARL for the five approaches studied in this work, and Figure 3 shows the respective control charts.

Table 3: Control limits, CLI and ARL of SH, SC, MC, WSD and MAD Control Charts.

	CLI	ARL
SH	0.4239	30
SC	0.3610	30
MC	0.2140	2.14
WSD	0.2957	15
MAD	0.4233	5

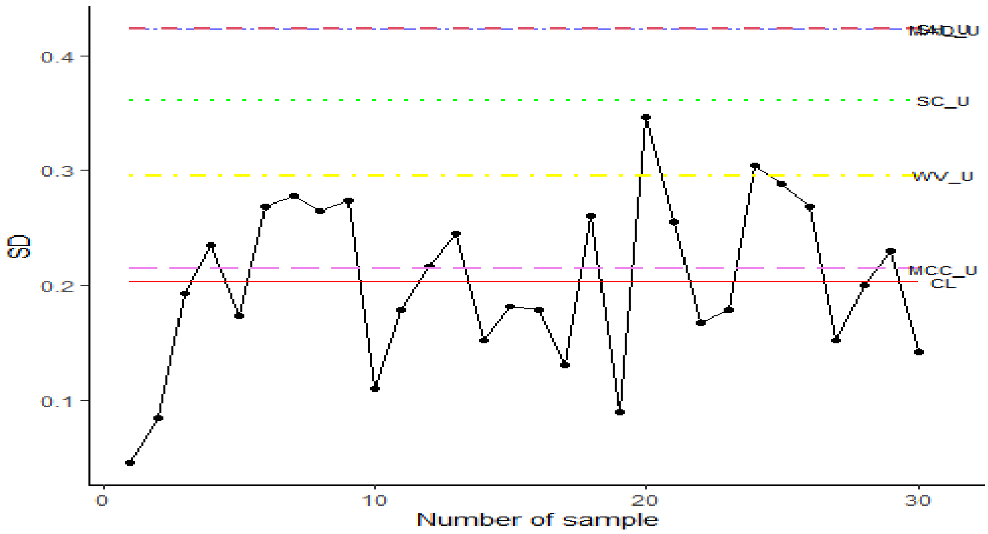

Figure 3: SH, SC, MC, WSD and MAD Control Charts for Thickness of Oil Seal Data

From the results presented in Table 3, the CLI and ARL of MC method are the smallest compared to SH, SC, WSD and MAD methods. Furthermore, It is evident in Figure 3 that in MC, there are 14 points out-of-control follow by WSD with 2 out-of-control points. This implies that MC method detects out-of-control faster and better than all other existing methods.

Conclusion

The use of MOILLD in this study has brought about a new Control Chart limits for standard deviation. Using CLI, ARL and SDRL as performance metrics, a simulation study was conducted to evaluate the performance of the proposed chart with SH, SC, WSD, and MAD control charts. The results of the MC control chart approach performed better and can identify out-of-control situations more quickly. The SC, WSD and MAD techniques would raise a false alarm in the absence of one since it was inappropriate for the simulated data. The real-life data results also showed that the MC method identify out-of-control quicker than other approaches. Control charts for the rival methodologies validate the findings. Thus, it can be inferred that the proposed charts will be better, and suitable for monitoring the dispersion of the data when the data are positively skewed and follow a MOILLD. Therefore, it is advised that users in the manufacturing and industry sectors implement the proposed approach, particularly in cases where the process data exhibit positive skewness.

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